

p189 22-27, 29 all

$$(22) \quad m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$(.44 \text{ kg})(3.30 \text{ ms}^{-1}) = (.44 \text{ kg})v_1' + (.22 \text{ kg})v_2'$$

$$\frac{1}{2} (.44 \text{ kg})(3.30 \text{ ms}^{-1})^2 = \frac{1}{2} (.44 \text{ kg})v_1'^2 + \frac{1}{2} (.22 \text{ kg})v_2'^2$$

$$1.452 = .44v_1' + .22v_2'$$

$$2.396 = .22v_1'^2 + .11v_2'^2$$

$$\rightarrow v_1' = \frac{1.452 - .22v_2'}{.44} = 3.3 - .5v_2'$$

$$2.396 = .22(3.3 - .5v_2')^2 + .11v_2'^2$$

$$2.396 = .22(10.89 - 3.3v_2' + .25v_2'^2) + .11v_2'^2$$

$$2.396 = 2.396 - .726v_2' + .055v_2'^2 + .11v_2'^2$$

$$.726v_2' = .165v_2'^2$$

$$\underline{v_2' = 4.4 \text{ ms}^{-1}}$$

$$v_1' = 3.3 - .5(v_2')$$

$$= 3.3 - .5(4.4 \text{ ms}^{-1})$$

$$\underline{v_1' = 1.1 \text{ ms}^{-1}}$$

$$(23) \quad m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$(0.45 \text{ kg})(3 \text{ ms}^{-1}) = (0.45 \text{ kg})v_1' + (0.9 \text{ kg})v_2'$$

$$(0.45 \text{ kg})(3 \text{ ms}^{-1})^2 = (0.45 \text{ kg})v_1'^2 + (0.9 \text{ kg})v_2'^2$$

$$1.35 = 0.45v_1' + 0.9v_2'$$

$$4.05 = 0.45v_1'^2 + 0.9v_2'^2$$

$$v_1' = \frac{1.35 - 0.9v_2'}{0.45} = 3 - 2v_2'$$

$$4.05 = 0.45(3 - 2v_2')^2 + 0.9v_2'^2$$

$$4.05 = 0.45(9 - 12v_2' + 4v_2'^2) + 0.9v_2'^2$$

$$4.05 = 4.05 - 5.4v_2' + 1.8v_2'^2 + 0.9v_2'^2$$

$$5.4v_2' = 2.7v_2'$$

$$\underline{v_2' = 2.00 \text{ ms}^{-1}} \quad (2.00 \text{ ms}^{-1} \text{ east})$$

$$v_1' = 3 - 2(v_2') = 3 - 2(2.00 \text{ ms}^{-1})$$

$$\underline{v_1' = -1.00 \text{ ms}^{-1}} \quad (1.00 \text{ ms}^{-1} \text{ west})$$

(24)

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$(2 \text{ ms}^{-1}) + (-3 \text{ ms}^{-1}) = v_1' + v_2'$$

$$(2 \text{ ms}^{-1})^2 + (-3 \text{ ms}^{-1})^2 = v_1'^2 + v_2'^2$$

$$-1 = v_1' + v_2'$$

$$13 = v_1'^2 + v_2'^2$$

$$\rightarrow v_1' = -1 - v_2'$$

$$13 = (-1 - v_2')^2 + v_2'^2$$

$$13 = 1 + 2v_2' + v_2'^2 + v_2'^2$$

$$0 = 2v_2'^2 + 2v_2' - 12$$

factor out the 2

$$0 = 2(v_2'^2 + v_2' - 6)$$

$$= 2(v_2' - 2)(v_2' + 3)$$

$$v_2' - 2 = 0$$

$$v_2' + 3 = 0$$

$$v_2' = 2$$

$$v_2' = -3$$

$$v_1 = -1 - v_2'$$

$$= -1 - 2$$

$$= -3$$

not possible

∴ The velocities are $\frac{2.0 \text{ m/s}}{v_2'}$ and $\frac{-3.0 \text{ m/s}}{v_1'}$

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$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$(.06 \text{ kg})(2.5 \text{ ms}^{-1}) + (.09 \text{ kg})(1.15 \text{ ms}^{-1}) = .06 \text{ kg } v_1' + .09 \text{ kg } v_2'$$

$$(.06 \text{ kg})(2.5 \text{ ms}^{-1})^2 + (.09 \text{ kg})(1.15 \text{ ms}^{-1})^2 = .06 \text{ kg } v_1'^2 + .09 \text{ kg } v_2'^2$$

$$.2535 = .06 v_1' + .09 v_2'$$

$$.4940 = .06 v_1'^2 + .09 v_2'^2$$

$$v_1' = \frac{.2535 - .09 v_2'}{.06} = 4.225 - 1.5 v_2'$$

$$.4940 = .06 (4.225 - 1.5 v_2')^2 + .09 v_2'^2$$

$$.4940 = .06 (17.85 - 12.675 v_2' + 2.25 v_2'^2) + .09 v_2'^2$$

$$.4940 = 1.071 - .7605 v_2' + .135 v_2'^2 + .09 v_2'^2$$

$$0 = .225 v_2'^2 - .7605 v_2' + .577$$

$$v_2' = \frac{.7605 \pm \sqrt{(-.7605)^2 - 4(.225)(.577)}}{2(.225)}$$

$$= \frac{.7605 \pm .243}{.45}$$

$$v_2' = 2.23 \text{ ms}^{-1}, 1.15 \text{ ms}^{-1}$$

this was ball 2's original velocity

so this will not be the solution.

$$v_1' = 4.225 - 1.5 v_2' = 4.225 - 1.5(2.23)$$

$$v_1' = 0.88 \text{ ms}^{-1}$$

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$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$(0.220 \text{ kg})(8.5 \text{ ms}^{-1}) = (0.22 \text{ kg})(-3.7 \text{ ms}^{-1}) + m_2 v_2'$$

$$(0.220 \text{ kg})(8.5 \text{ ms}^{-1})^2 = (0.22 \text{ kg})(-3.7 \text{ ms}^{-1})^2 + m_2 v_2'^2$$

$$1.87 = -0.814 + m_2 v_2'$$

$$15.895 = 3.012 + m_2 v_2'^2$$

$$m_2 = \frac{1.87 + 0.814}{v_2'} = \frac{2.684}{v_2'}$$

$$15.895 = 3.012 + \left(\frac{2.684}{v_2'} \right) v_2'^2$$

$$v_2' = 4.8 \text{ ms}^{-1}$$

$$m_2 = \frac{2.684}{v_2'} = \frac{2.684}{4.8 \text{ ms}^{-1}}$$

$$m_2 = 0.56 \text{ kg}$$

(27)

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$(450 \text{ kg})(4.5 \text{ ms}^{-1}) + (550 \text{ kg})(3.7 \text{ ms}^{-1}) = (450 \text{ kg}) v_A' + (550 \text{ kg}) v_B'$$
$$(450 \text{ kg})(4.5 \text{ ms}^{-1})^2 + (550 \text{ kg})(3.7 \text{ ms}^{-1})^2 = (450 \text{ kg}) v_A'^2 + (550 \text{ kg}) v_B'^2$$

$$4060 = 450 v_A' + 550 v_B'$$
$$16642 = 450 v_A'^2 + 550 v_B'^2$$
$$v_A' = \frac{4060 - 550 v_B'}{450} = 9.02 - 1.22 v_B'$$

$$16642 = 450(9.02 - 1.22 v_B')^2 + 550 v_B'^2$$
$$16642 = 450(81.4 - 22.01 v_B' + 1.49 v_B'^2) + 550 v_B'^2$$
$$16642 = 36630 - 9904.5 v_B' + 670.5 v_B'^2 + 550 v_B'^2$$
$$0 = 19988 - 9904.5 v_B' + 1220.5 v_B'^2$$

divide everything by 1220.5 to simplify numbers.

$$0 = 16.38 - 8.12 v_B' + v_B'^2$$

$$v_B' = \frac{8.12 \pm \sqrt{(8.12)^2 - 4(1)(16.38)}}{2(1)}$$
$$= \frac{8.12 \pm 0.6437}{2}$$

$$v_B' = 4.38 \text{ ms}^{-1}, \quad \overbrace{3.74 \text{ ms}^{-1}}$$

this is the speed we started with.

$$v_A' = 9.02 - 1.22 v_B' = 9.02 - 1.22(4.38)$$

$$v_A' = 3.68 \text{ ms}^{-1}$$

$$(29) \quad mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ ms}^{-2})(.3 \text{ m})} = 2.43 \text{ ms}^{-1}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m_1 = M$$

$$m_2 = \frac{M}{2}$$

$$Mv_1 = Mv_1' + \frac{Mv_2'}{2}$$

$$Mv_1^2 = Mv_1'^2 + \frac{Mv_2'^2}{2}$$

$$2v_1 = 2v_1' + v_2'$$

$$2v_1^2 = 2v_1'^2 + v_2'^2$$

$$2(2.43 \text{ ms}^{-1}) = 2v_1' + v_2'$$

$$2(2.43 \text{ ms}^{-1})^2 = 2v_1'^2 + v_2'^2$$

$$4.86 = 2v_1' + v_2'$$

$$11.81 = 2v_1'^2 + v_2'^2$$

$$\rightarrow v_2' = 4.86 - 2v_1'$$

$$11.81 = 2v_1'^2 + (4.86 - 2v_1')^2$$

$$11.81 = 2v_1'^2 + 23.62 - 19.44v_1' + 4v_1'^2$$

$$0 = 6v_1'^2 - 19.44v_1' + 11.81$$

divide by 6 for easier solution

$$0 = v_1'^2 - 3.24v_1' + 1.97$$

$$v_1' = \frac{3.24 \pm \sqrt{(-3.24)^2 - 4(1)(1.97)}}{2}$$

$$= \frac{3.24 \pm 1.81}{2}$$

$$v_1' = 0.72 \text{ ms}^{-1}, \quad \cancel{2.53 \text{ ms}^{-1}} \quad (\text{speed cannot increase})$$

$$v_2' = 4.86 - 2v_1' = 4.86 - 2(0.72 \text{ ms}^{-1})$$

$$v_2' = 3.42 \text{ ms}^{-1}$$

29 cont'd

Mass M ($v_1' = 0.72 \text{ ms}^{-1}$)

$$\begin{array}{c} \underline{x} \\ v_x = 0.72 \text{ ms}^{-1} \end{array}$$

$$s_x = ?$$

$$t = 0.428 \text{ s}$$

$$s_x = vt$$

$$= (0.72 \text{ ms}^{-1})(0.428 \text{ s})$$

$$= \underline{0.31 \text{ m}}$$

$$\begin{array}{c} \underline{y} \\ u_y = 0 \end{array}$$

$$s_y = -.9 \text{ m}$$

$$a = -9.81 \text{ ms}^{-2}$$

$$t = \underline{\hspace{2cm}}$$

$$s = ut + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2(-.9 \text{ m})}{-9.81 \text{ ms}^{-2}}} = 0.428 \text{ s}$$

Mass m ($v_2' = 3.42 \text{ ms}^{-1}$)

$$\begin{array}{c} \underline{x} \\ v_x = 3.42 \text{ ms}^{-1} \end{array}$$

$$s_x = ?$$

$$t = 0.428 \text{ s}$$

$$s_x = vt$$

$$= (3.42 \text{ ms}^{-1})(0.428 \text{ s})$$

$$= \underline{1.5 \text{ m}}$$

$$\begin{array}{c} \underline{y} \\ u_y = 0 \end{array}$$

$$s_y = -.9 \text{ m}$$

$$a = -9.81 \text{ ms}^{-2}$$

$$t = \underline{\hspace{2cm}}$$

$$s = ut + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2(-.9 \text{ m})}{-9.81 \text{ ms}^{-2}}} = 0.428 \text{ s}$$